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Beyond Core Knowledge: Natural Geometry

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Abstract

For many centuries, philosophers and scientists have pondered the origins and nature of human intuitions about the properties of points, lines, and figures on the Euclidean plane, with most hypothesizing that a system of Euclidean concepts either is innate or is assembled by general learning processes. Recent research from cognitive and developmental psychology, cognitive anthropology, animal cognition, and cognitive neuroscience suggests a different view. Knowledge of geometry may be founded on at least two distinct, evolutionarily ancient, core cognitive systems for representing the shapes of large-scale, navigable surface layouts and of small-scale, movable forms and objects. Each of these systems applies to some but not all perceptible arrays and captures some but not all of the three fundamental Euclidean relationships of *distance* (or *length*), *angle*, and *direction* (or *sense*). Like natural number (Carey, 2009), Euclidean geometry may be constructed through the productive combination of representations from these core systems, through the use of uniquely human symbolic systems.

Keywords: Spatial cognition; Cognitive development; Conceptual change; Form perception; Navigation

1. Introduction

Of all our abstract conceptual systems, two of the simplest are the positive integers—the set of concepts composing the system of *natural number*—and the points, lines, and figures of the Euclidean plane—the set of concepts composing the system of *natural geometry* (Descartes, 1637/2001). Despite their simplicity, however, natural number concepts do not appear to come so naturally to humans. Carey (2009), whom this study honors, argues that the positive integers are constructed from a set of core cognitive systems

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that are considerably less general or powerful: a system for representing exactly 1, 2, or 3 numerically distinct individuals, on one hand, and a system for representing and operating on approximate cardinal values, on the other. Moreover, the construction of integers depends in part on culturally variable counting devices. Natural number may be therefore partly a product of human culture, built on a foundation of core systems that emerge in infancy, guide the reasoning of adults in all cultures, and are shared with other animals.

What about natural geometry? For 2,500 years, the system of geometry that has appeared most natural to human adults is Euclidean plane geometry: a formal system for characterizing two-dimensional (2D) shapes in accord with the *distance*, *angle*, and *directional* relationships among their parts. In Euclidean geometry, all forms that differ from one another have parts that contrast on one or more of these properties; conversely, two forms whose points and lines are characterized by the same distance, angle, and directional relationships are congruent (see Fig. 1). The geometrical equivalence of two forms with the same length, angle, and sense relations is so immediately apparent to the human mind that philosophers and mathematicians believed for millennia that Euclidean geometry constituted the only logically possible geometric system (Hatfield, 1990; Kline, 1972). Although this view was overturned more than two centuries ago, Euclidean geometry continues to be the first system of formal geometry that students learn, and the system whose principles accord best with human intuition.

What are the sources of Euclidean geometrical intuitions? Philosophers from Plato (ca. 380 B.C 1949) to Descartes (1637/2001) to Kant (1781[2003]) have argued that Euclidean geometry comes naturally to the human mind, even to the minds of humans who lack all instruction in mathematics or experience in a locally Euclidean world, for how could a creature who lacked this system of spatial concepts ever gain the kinds of spatial experiences that would support it (see Hatfield, 1990 for discussion)? More recently, some evolu-

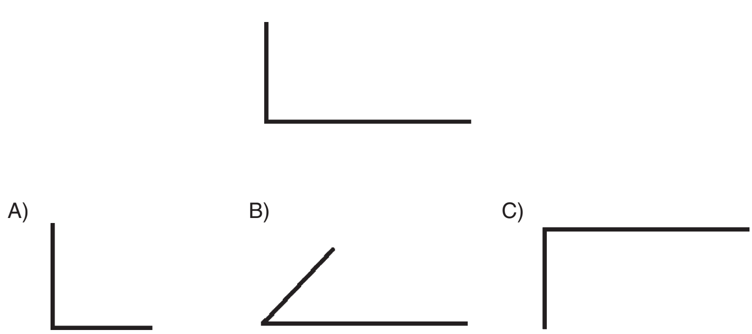

Fig. 1. A geometrical theory can be fully characterized by the set of transformations that leave the properties of figures invariant (Klein, 1893). In Euclidean plane geometry, two forms are identical if they can be made to coincide through a rigid displacement (rotation and/or translation) on the plane. Forms that cannot be made to coincide differ in one or more of the properties of (A) *distance* (the lengths of parts and distances between them), (B) *angle* (the orientations at which parts meet), and (C) *sense* (the left–right directions of parts with respect to one another).

tionary biologists and psychologists have posited innate cognitive mechanisms for capturing locally Euclidean geometric relationships, reasoning that navigating organisms cannot afford to lose their way while learning those relationships by trial and error (Gallistel, 1990). And some computer scientists, working in robotics or computer vision, have built Euclidean principles into the systems by which autonomously moving robots map the environment (e.g., Gee, Chekhlov, Calway, & Mayol-Cuevas, 2008) or recognize objects (e.g., Marr, 1982; see also Biederman, 1987), as Euclidean geometry provides for economical descriptions of surfaces and objects.

Countering these arguments are proposals that all of geometry is learned by processes of association or of rapid, adaptive learning, applied to the spatially structured world projected to our senses (Berkeley, 1709/1975; von Helmholtz, 1885/1962). Consistent with these views, animals learn to use arbitrary landmarks as guides to environmental locations (for review, see Cheng & Newcombe, 2005), children's spatially guided behavior shows a regular increase in precision with growth and experience (e.g., Spencer & Hund, 2003), and adults' navigation is systematically altered by experience (e.g., Rieser, Hill, & Taylor, 1992).

What, then, is the nature of human knowledge of geometry, and how does this knowledge arise and develop? Here, we offer a hypothesis in the spirit of Carey (2009) that contrasts with both families of proposals sketched earlier. Like natural number, natural geometry is founded on at least two evolutionarily ancient, early developing, and cross-culturally universal cognitive systems that capture abstract information about the shape of the surrounding world: two *core systems of geometry*. Nevertheless, each system is limited: It captures only a subset of the properties encompassed by Euclidean geometry, and it applies only to a subset of the perceptible entities to which human adults give shape descriptions. Children go beyond these limits and construct a new system of geometric representation that is more complete and general, by combining productively the representations delivered by these two systems. This productive, combinatorial process, we suggest, depends in part on uniquely human, culturally variable artifacts: pictures, models, and maps. Thus, like the system of number, the system of geometry that feels most natural to educated adults is a hard-won cognitive achievement, constructed by children as they engage with the symbol systems of their culture.

2. Euclidean cognitive maps: Core geometry for action?

Before turning to this hypothesis, we briefly consider a different family of hypotheses concerning the origins of human geometrical intuitions. It is possible that geometrical intuitions emerge not from systems for representing the shapes of the surrounding world but from systems for representing the spatial structure of one's own actions. Because all actions involve motion through space, effective actions require that agents direct their eyes, hands, and bodies to the right places. Once actions are coordinated appropriately, agents might therefore be able to use the spatial information in their actions to deduce the spatial properties of the surrounding environment. Some action-based theories of spatial knowledge give a central role to learning and experience; they posit that animals and humans construct Euclidean representations of the environment through active experimentation and the development of motor skills (e.g., von Helmholtz, 1885/1962; Piaget, 1952; Spencer, Smith, & Thelen, 2001). Other action-based theories propose that animals and humans are endowed with an innate capacity to represent the Euclidean properties of their actions, and they use this capacity to deduce the Euclidean properties of external objects and scenes (Descartes, 1637/2001; Gallistel, 1990; Landau, Gleitman, & Spelke, 1981; Tolman, 1948). On both sets of views, natural geometry originates in an ability to use the spatial structure of one's own actions to form a ''cognitive map'' of the environment.

Tests of the cognitive map hypothesis date back to Tolman (1948), whose studies of latent learning and shortcut behavior suggested that rats who move through an environment record the locations of the places through which they travel in a geocentric, Euclidean coordinate system. This hypothesis was bolstered by studies of triangle completion in navigating animals from insects (Gould, 1986) to humans (Loomis, Klatzky, Golledge, & Philbeck, 1999). Cognitive maps were given a neural interpretation through seminal studies of the patterns of activity of pyramidal neurons in the rat hippocampus, whose firing rates depended on the rat's position in the environment, regardless of whether that environment could be seen (O'Keefe & Nadel, 1978).

Nevertheless, none of these studies conclusively showed that animals form Euclidean maps of their navigable environment. Indeed, further studies of navigation provided evidence that insects, rats, and even humans often fail to do so and instead find their way by learning and reproducing a series of routes between significant locations (Cartwright & Collett, 1982; Foo, Warren, Duchon, & Tarr, 2005; Restle, 1957; Wehner & Menzel, 1990). Particularly striking evidence against Euclidean maps has been obtained through recent studies of human navigation in virtual reality (Foo et al., 2005; Foo, Warren, Duchaine, & Tarr, 2007). In these studies, college students were allowed to move through environments on paths whose distance and angular relationships either preserved or violated the laws of Euclidean geometry. Strikingly, students navigated as effectively in non-Euclidean as in Euclidean environments, and their navigation by purely Euclidean relationships was highly inaccurate. Because these studies were performed on educated human adults, their findings cast doubt on all theories that place actions at the center of human Euclidean intuitions, whether nativist or empiricist in spirit. Even adults who have spent a lifetime navigating in locally Euclidean space, and who have learned formal Euclidean geometry, fail to organize their paths of locomotion into a Euclidean cognitive map.

If the concepts of Euclidean geometry do not originate in our coordinated action, then what are their sources? ''*Geo-metry*'' is the measurement of the Earth. Consistent with this meaning, research across the spectrum of the cognitive sciences provides evidence for at least two systems by which animals and humans measure their perceivable surroundings. One system applies to the large-scale spatial layout and guides navigation. The other system applies to small-scale forms and allows for recognition and categorization of objects by their shapes. We consider these two systems in turn.

3. Core geometry for navigation

Like other functions of evolutionary importance, navigation depends on multiple mechanisms. Some of these mechanisms are not guided by Euclidean information, such as the mechanisms at play in the action tasks described earlier (Wehner & Menzel, 1990). Some of these mechanisms are species specific, such as the celestial navigation of migratory birds (Emlen, 1970). Some mechanisms involve little or no analysis of geometry, such as the following of odor trails by ants (Carthy, 1951). One mechanism of navigation, however, is both widespread across animals, including humans, and centrally focused on environmental geometry. As humans and other animals navigate, they represent both *distances* and *directional relationships* on the extended surfaces that bound the navigable layout (Doeller & Burgess, 2008; Lever, Wills, Cacucci, Burgess, & O'Keefe, 2002). The existence and properties of these representations are revealed when animals lose their orientation and must draw on memory for the positions of these surfaces to reorient themselves (Cheng & Gallistel, 1984). The cognitive system that accomplishes this feat is the first of our two proposed core systems of geometry, so we will describe it in some detail.

Cheng (1986) was the first to discover that rats recover their orientation by analyzing the shape of their surroundings. When rats explored a rectangular room in which food was buried and then were disoriented by slow turning in the dark, they used the lengths and relative directions of the room's walls to reorient themselves and therefore searched for the food at the two locations that were congruent with the room's geometry (e.g., at a corner to the *left* of a *long* wall; Fig. 2). Subsequent research revealed that the capacity to reorient by the shape of the borders of the environment is found in animals as distant as humans (Hermer & Spelke, 1994; Lourenco & Huttenlocher, 2006) and ants (Wystrach & Beugnon, 2009).[1] Importantly, the ability develops in animals independently of experience in a geometrically structured layout: Chicks and fish who were raised since hatching in a geometrically uninformative, circular environment reoriented by the shape of a rectangular environment the first time they encountered it, and they did this as reliably as chicks or fish who were experienced at navigating by geometry (Brown, Spetch, & Hurd, 2007; Chiandetti & Vallortigara, 2008). In contrast, disoriented animals' use of nongeometric properties such as surface brightness or texture is highly influenced by experience, both in controlled-reared fish (Brown et al., 2007) and in mice who are trained to use surface features to locate objects (Twyman, Newcombe, & Gould, 2009). Finally, sensitivity to geometry is shown across a wide variety of navigation tasks, in oriented as well as disoriented animals or humans tested by a diverse set of behavioral and neurophysiological methods (see Cheng & Newcombe, 2005 for review). It is encoded automatically, independently of processes for encoding other features of the environment such as landmark objects (Doeller & Burgess, 2008), by a distinct neuronal network that includes the hippocampus (Doeller, King, & Burgess, 2008) and surrounding cortical areas (Epstein, 2008; Solstad, Boccara, Kropff, Moser, & Moser, 2008).

Nevertheless, the system for encoding surface layout geometry fails as a system of Euclidean geometry in two ways. First, Euclidean geometry can be applied to any perceptible objects, but the core geometric navigation system is more limited: Navigating animals encode the shapes of the extended surfaces that form the borders of the traversable layout,

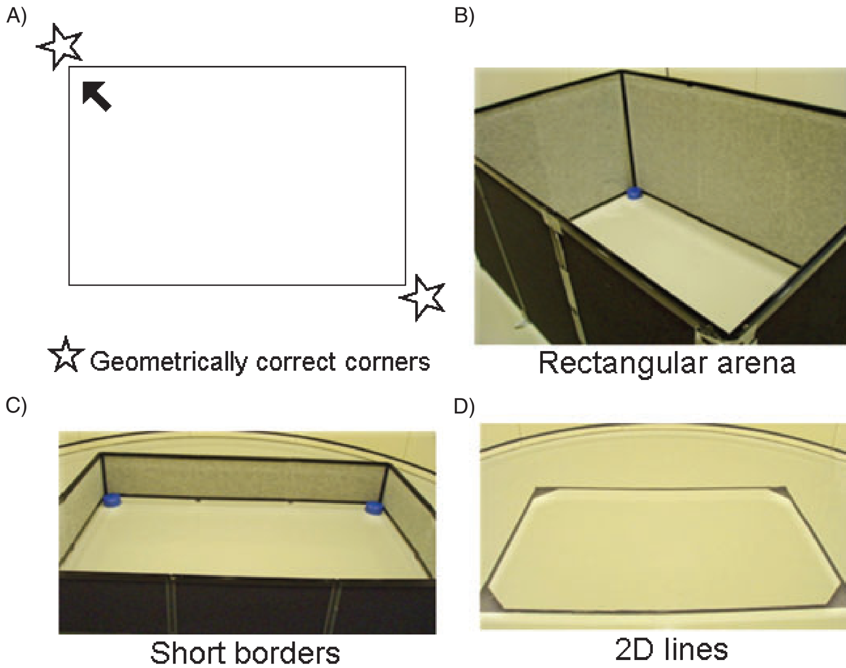

Fig. 2. Examples of rectangular arrays and environments. In (A), the arrow symbolizes the correct goal location in the schematic diagram; the stars symbolize the two geometrically correct locations—the correct corner and its diagonally opposite twin. In a rectangular arena with (B) tall walls (rats: Cheng, 1986; children: Hermer & Spelke, 1994) or (C) short borders (children: Lee & Spelke, 2008), search is limited to the two geometrically correct corners. When the walls or borders are replaced by 2D rectangular lines (D: Lee & Spelke, 2008), children search all four corners equally.

but they fail to encode the shapes of surface markings or landmark objects. Cheng and Gallistel's (1984) rats, for example, reoriented by the shape of their chamber but not by the shapes of the visual forms that decorated the corners of that chamber. In later studies, 4-year-old children reoriented themselves in accord with the shape of a rectangular room, a rectangular array of walls with an open ceiling, and a rectangular array of borders that are low enough to step over, but they failed to reorient by the shape of a rectangular array of 3D objects, large columns, or black lines on the floor (Lee & Spelke, 2008; see also Gouteux & Spelke, 2001; Fig. 2). Importantly, children's failure to reorient by the shapes of arrays of objects or visual forms does not stem from a failure to detect or attend to those objects and forms: When an object was hidden at a corner of a 2D black rectangle, for example, children searched directly and exclusively at the rectangle's four corners, showing that they detected the rectangle and were aware of its relevance for the search task. Nevertheless, children failed to use the length and sense relations in the rectangle to limit their search to the two geometrically congruent corners, as they do when they are presented with a geometrically identical array of extended surfaces.

Even more dramatic evidence for a dissociation between detecting and reorienting by 2D geometry comes from experiments by Huttenlocher and Lourenco (2007). They investigated toddlers' reorientation in a square room whose alternating walls could be distinguished in various ways. First, they showed that children failed to reorient by a color difference between the opposite walls of the chamber: a conceptual replication of Cheng's (1986) original findings. When opposite wall pairs were red and blue and an object was hidden at one of the corners with the red wall on the left, disoriented children searched the four corners equally, irrespective of their lateral relationships (see also Dessalegn & Landau, 2008; Lourenco, Addy, & Huttenlocher, 2009), although children do reorient in a square room when opposite walls differ in brightness (Nardini, Atkinson, & Burgess, 2008). Next, Huttenlocher and Lourenco showed that children successfully reorient by differences in wall texture size and density: When opposite walls were covered with large and sparse versus small and dense circles, children confined their search to the two corners with the appropriate textural and directional relationships (e.g., searching only corners with the smaller, denser circles on the left). Thus, children detected the circles on the walls and used them for some purposes. Finally, children's reorientation was tested in a square room whose walls differed in geometric form (one pair of opposite walls was covered with circles and the other with crosses) or in which the circular, patterned walls alternated with walls containing no pattern. The geometry of surface markings had no effect on children's reorientation in either of these conditions, despite children's clear ability to detect these markings.[2]

The tendency of children and animals to encode automatically the shape of the surface layout, but not the shapes of objects or patterns, has puzzled both experimental psychologists and behavioral biologists. Computational studies of navigation, however, may shed light on this tendency. In natural environments, objects and small patterns often have identical or nearly identical twins: One leaf or bush may look much like another. Moreover, objects often are moveable. Thus, navigating robots that record their position with respect to objects or small-scale visual patterns are apt to confuse locations containing similar objects (Milford & Wyeth, 2008) and to fail to recognize locations after one or more objects has moved (Silveira, Malis, & Rives, 2008). Moreover, most natural scenes contain a rich array of objects and surfaces markings, whose detail can only be captured by recording a great deal of information. Because surfaces tend to form a geometrically unique configuration, to persist over time, and to be smooth, however, the positions of extended surfaces can be recorded more effectively and economically. For example, the 3D coordinates of just three points suffice to specify the distance and orientation of a planar surface (see Gee et al., 2008). Thus, a reorientation system that focuses only on the geometry of extended surfaces may form representations that are distinctive, robust over object motion, and economical.

The geometric reorientation system has a second limit: It fails to capture the Euclidean geometrical relationship of *angle*. Hupbach and Nadel (2005) tested for effects of angle by disorienting children in a rhombic environment with four equal-length walls arranged to form two obtuse and two acute angles. When an object was hidden at one corner of the rhombus, disoriented children searched the four corners equally, despite their markedly different angular relations and aspect ratio. This limit, which has been replicated in a variety of different environments (S. A. Lee & E. S. Spelke, unpublished data) and persists in children up to 4 years of age (Hupbach & Nadel, 2005), reveals that children reorient by *distance* and *direction* but not by *angle*. It provides the strongest evidence against the thesis that reorientation is guided by a general representation of Euclidean geometry.[3]

In summary, studies of navigation provide evidence for a core system of geometric representation that guides navigation both in animals and in young children. The system is truly geometric in three respects. First, it captures shape relationships abstractly: Animals navigate by the shape of a chamber regardless of whether the chamber is visible (Quirk, Muller, & Kubie, 1990) and over dramatic changes in the chamber's color, texture, and material composition (Lever et al., 2002). Second, it preserves information for Euclidean *distance* and left–right *direction*: two fundamental properties of Euclidean geometry. Third, it supports inferences about the orientation of the self and the locations of objects and significant places. Nevertheless, the system fails to apply to the simplest and most prototypical objects of Euclidean geometry: 2D surface markings. Moreover, it fails to capture the central Euclidean property of *angle*. Thus, the core navigation system is not the complete system of ''natural geometry'' envisaged by Plato, Descartes, or Kant.

4. Core geometry for form perception and object shape description

Although animals and children are strikingly oblivious to surface markings and angular relationships in the large-scale navigable layout, they are highly sensitive to surface markings and angular relationships in small pictures and objects. This conclusion, supported by many decades of experiments on form perception and object recognition in animals and human infants (see Gibson, 1969; for a classic review of the earlier literature), is reinforced by recent experiments probing the development of sensitivity to length, angle, and direction in visual forms (Izard & Spelke, 2009). The experiments, conducted on adults and on children ranging from 4 to 10 years of age, revealed a markedly different pattern of performance from that found in studies of navigation. At all ages, children and adults detected angle and length relationships with relative ease, but they failed to detect directional relationships until adolescence.

The experiment used a deviant detection paradigm (after Dehaene, Izard, Pica, & Spelke, 2006) in which five forms in the shape of an L shared a geometric property that a sixth L-shaped form lacked. Children were presented with the six forms in a random arrangement and at random orientations, and their task was to detect the geometrical deviant. On different trials, the deviant form differed from the other forms in line length, angle, or sense (Fig. 3A). On pure trials, all the forms were otherwise identical; on interference trials, the forms varied along a second irrelevant dimension. A comparison of the latter trials to the pure trials served to assess whether length, angle, or sense was processed automatically and interfered with the detection of the relevant dimension.

There were two main findings. First, participants of all ages showed highest sensitivity to angular and length relations and lowest sensitivity to the sense relation that distinguishes a form from its mirror image (Fig. 3B). Second, variations in angle and length interfered with one another and with processing of sense, but variations in sense had no effect on processing

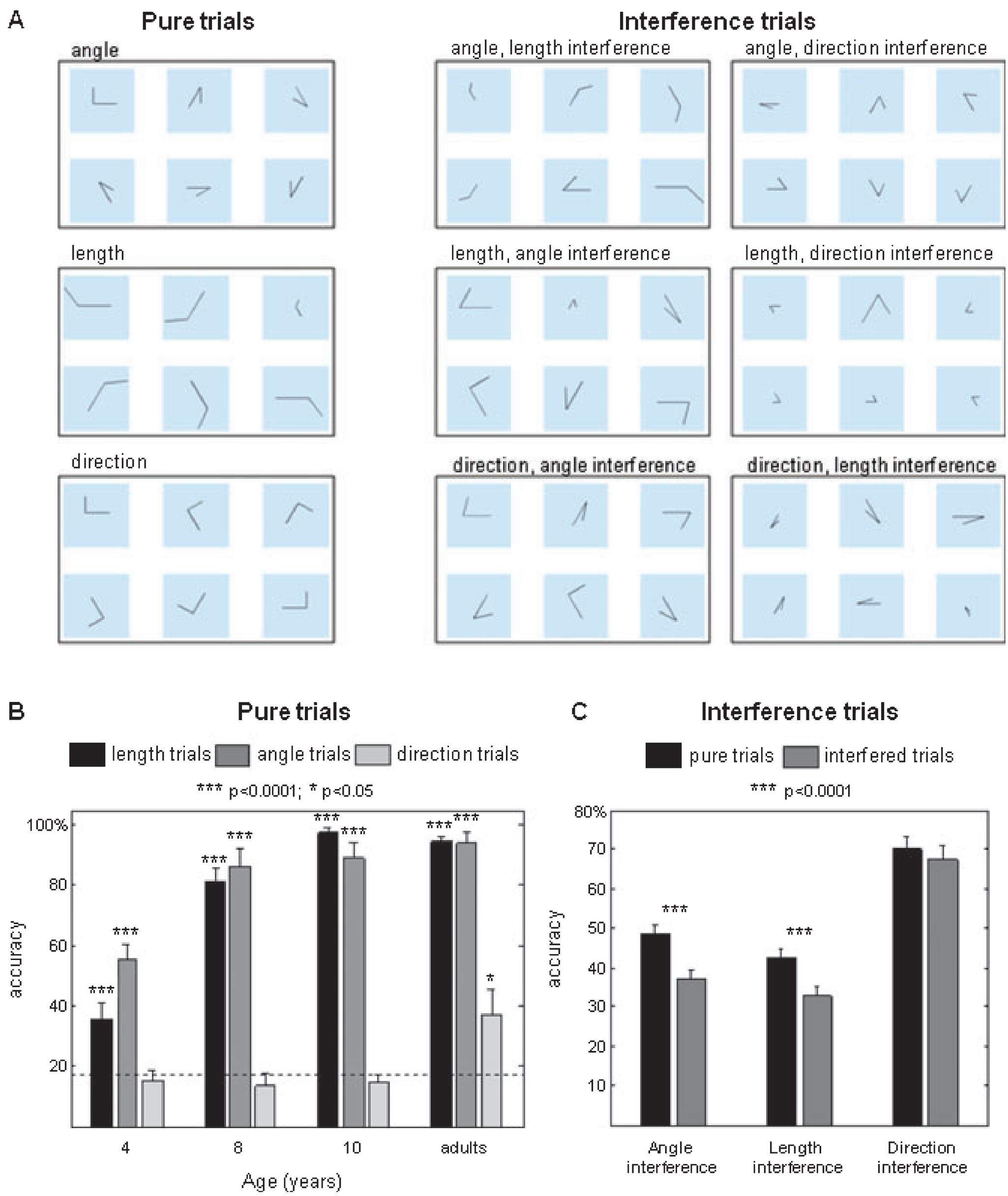

Fig. 3. Sensitivity to geometry in visual forms. (A) Example displays. On different trials, the deviant form was distinguished by its length, angle, or sense relations; the other forms were either alike in all other geometric relations (pure trials) or varied on a second geometric relation (interference trials). (B) Performance of each age group on pure trials. At all ages, performance is better on angle and length deviants than on sense deviants. (C) Effects of interfering variation in length, angle, or sense on detection of the other two geometric properties. From left to right, pairs of bars depict the performance on distance and sense trials, angle and sense trials, and distance and angle trials. Performance was impaired by irrelevant variation in angle or distance but not sense (gray bars), relative to performance on trials lacking that variation (black bars).

of angle or length (Fig. 3C). At all ages, therefore, sense is difficult to detect and easy to ignore.

As a large literature documents, adults' ability to distinguish a form or 3D object from its mirror image often requires the application of a mental rotation to align the stimuli (Cooper & Shepard, 1973). After rotation, two visual forms or objects can be compared directly by a process of template matching, with no need to represent their abstract sense relations. Mental rotation therefore appears as a strategy that is applied to compensate for the absence of abstract, orientation-independent representations of sense. In contrast, the deviant detection study showed that angle and length relations are processed reliably and rapidly in figures that vary in orientation. Processing of angle and length likely does not elicit the processes of mental rotation that are required for the discrimination of mirror images.

The asymmetry between the detection of length and angle, on one hand, and sense on the other, may trace back to infancy. Decades of experiments have investigated sensitivity to angle and length in human infants, sometimes as early as a few hours after birth (Slater, Mattock, Brown, & Bremner, 1991). In one set of studies, infants were habituated to two lines crossing at a constant angle, presented at a number of different orientations. Then they were tested with new displays, consisting of the same shape in a previously unseen orientation, or of a shape that differed either by angle or length alone. Infants generalized habituation to the former displays and looked longer at the latter ones, suggesting that they are sensitive to variations of angle (Schwartz & Day, 1979; Slater et al., 1991). Similar experiments showed that infants are also sensitive to variations in length (Newcombe, Huttenlocher, & Learmonth, 1999). In contrast, studies probing infants' detection of sense have yielded mixed findings: Infants failed to distinguish forms from their mirror images in some experiments (Lourenco & Huttenlocher, 2008), and only subsets of infants succeeded in other experiments (Moore & Johnson, 2008; Quinn & Liben, 2008). However, sensitivity to direction was tested with different displays than those used to assess sensitivity to angle and length, raising the possibility that extraneous stimulus differences, rather than differences in the geometric properties, produced the differing outcomes. Moreover, none of the studies reveal whether the infants extracted geometric information in an orientation-invariant manner or mentally rotated the displays into alignment. Infants' sensitivity to the Euclidean properties of visual forms therefore merits further study.

Returning to older children and adults, the findings of studies of 2D form perception complement the findings of a large body of research on 3D object recognition. Many aspects of object recognition are subject to controversy, including the fundamental properties that object representations capture (e.g., surfaces vs. volumes) and the frames of reference within which those properties are represented (e.g., view-based vs. viewpoint-invariant; see Biederman & Cooper, 2009; Riesenhuber & Poggio, 2000). It is widely agreed, however, that objects are best recognized by their shapes, beginning in early childhood (Smith, Jones, Landau, Gershkoff-Stowe, & Samuelson, 2002) and continuing through adulthood (Biederman, 1987; Marr, 1982; Warrington & Taylor, 1978). Adults across cultures categorize both 3D object shapes and 2D forms with respect to basic Euclidean properties such as the presence of straight edges and parallel surfaces (Biederman, Yue, & Davidoff, 2009; Dehaene et al., 2006). In both behavioral and neuroimaging studies, moreover, shape-based object recognition has been found to be invariant both over a wide range of scales and over reflection (Biederman & Cooper, 2009), providing evidence for sensitivity to differences in length and angular relationships but not to the directional relationship that distinguishes a shape from its mirror image. Finally, object shape is processed by dedicated regions in the lateral occipital and temporal cortex of the brain (Grill-Spector, Golarai, & Gabrieli, 2008; Reddy & Kanwisher, 2006). These regions respond to the shapes both of 3D objects and of 2D forms, in humans (Kourtzi & Kanwisher, 2001) and in nonhuman primates (Kriegeskorte et al., 2008; Tanaka, 1996; Yamane, Carlson, Bowman, Wang, & Connor, 2008), further suggesting that common cognitive mechanisms underlie perception of the shapes of 2D visual forms and of 3D manipulable objects.

In summary, research provides evidence for a core system for representing the shapes of movable, manipulable objects. This system shows qualitative continuity over human development (Izard & Spelke, 2009) and across cultures (Dehaene et al., 2006). It captures abstract geometric information, representing object shapes over considerable variations in orientation, substance, and texture. It highlights two fundamental properties of Euclidean geometry, *length* and *angle*. Yet this system also falls short of full Euclidean geometry in two respects. First, it fails to apply to the large-scale, navigable layout, as evidenced by children's failure to respond to angle in a rhombic room, and by animals' and children's failure to use the different patterns on walls or corners to reorient themselves. Second, it captures Euclidean *distance* and *angle* but not *sense*, and therefore fails to distinguish a form from its mirror image. Core processes of visual form analysis are also evidently not the sole foundation for Euclidean geometry.

5. Two core systems of geometry

Fig. 4a summarizes the contrasting properties of the two proposed systems. Note that neither system alone applies to entities both small and large, manipulable and navigable. Moreover, neither system captures all the properties of Euclidean geometry. Finally, neither system is simply more sensitive nor more widely applicable than the other; instead, they show qualitatively different specializations and limits. These observations suggest that separate systems underlie the processing of geometry in large-scale environmental spatial layouts and in small-scale objects and forms.

Psychologists and neuroscientists have long recognized that dissociable shape-processing systems support the navigation and visual form analysis. In studies of human adults using functional brain imaging, the regions that are most activated by the shapes of forms and objects (e.g., lateral occipital complex; Grill-Spector et al., 2008) are located far from the regions that process geometric information for navigation (e.g., the hippocampus and parahippocampal cortex, as well as regions of occipital and parietal cortex; Burgess, Jeffery, & O'Keefe, 1999; Epstein & Kanwisher, 1998; Landau & Lakusta, 2009). In neurophysiological studies of animals, distinct brain systems have been found to process distinct kinds of spatial information in order to recognize objects, on one hand, and locate those objects in the larger environmental layout, on the other (Mishkin, Ungerleider, & Macko, 1983).

A. Core systems of Geometry

	Length / Distance	Angle	Sense / Direction
Navigating 3D spatial layout	√	––	√
Analyzing 2D visual forms	√	√	––

B. Core Systems of Number

	1 vs. 2	4 vs. 8	2 vs. 3
Tracking individuals	√	––	√
Comparing sets	√	√	––

Fig. 4. Schematic depiction of the core systems of geometry (A) and number (B).

In studies of humans with cortical brain damage, selective impairments to these different kinds of spatial processing have been observed (Goodale & Milner, 1992). This neural and cognitive distinction is reflected in human languages, which use precise shape and angle information to specify the referents of object names, but crude distance and sense information to specify the referents of spatial prepositions (Landau & Jackendoff, 1993; Landau & Lakusta, 2009). Thus, a wealth of research is consistent with the findings from studies of young children.

If these two core systems are distinct and limited, however, their union would have considerably more generality and power. Together, the two systems capture all of the fundamental properties of Euclidean geometry: *distance*, *angle*, and *directional relationships*. Together, moreover, they allow for a common description of small-scale objects and of large-scale spatial layouts. Euclidean geometry may not be immediately available to a child or animal endowed with these two systems, but it could be constructed by productive combination of their outputs.

That is precisely the lesson of studies of the construction of natural number (see Carey, 2009; Feigenson, Dehaene, & Spelke, 2004). Natural number is founded on two core systems of representation, each with true but limited numerical content (Fig. 4b). One system serves to represent numerically distinct individuals, supports the concept *one*, and allows for the operation of *adding one* to an array, but it includes no explicit, summary representations of other cardinal values (such as *two*) and has a capacity limit of about 3 individuals. The second system serves to represent sets and their approximate cardinal values, supports concepts such as *about eight*, and allows for operations of comparison and arithmetic on those concepts, but it has no successor function (*one more*) and is subject to a ratio limit on precision that rises from about .33 at birth to about .88 at maturity.

A child with access to the two core number systems could, in principle, construct the system of natural number concepts by combining together these concepts and operations. In practice, this construction takes many years, and it involves mastery of the counting system used in the child's culture (Carey, 2009; Wynn, 1990). Although much remains to be learned about the steps by which children construct natural number from these core systems, there is evidence that experience in a numerate culture, with devices for symbolizing and operating on exact numbers, is critical for the development and exercise of natural number concepts. Children appear to acquire natural number concepts when they master their culture's counting procedure (Carey, 2009). Moreover, adults who live in a culture lacking a verbal-counting procedure typically show only limited abilities to represent large, exact cardinal values (Frank, Everett, Fedorenko, & Gibson, 2008; Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004; although see Butterworth, Reeve, Reynolds, & Lloyd, 2008). In numerate societies, symbolic systems other than language also support numerical reasoning (see Dehaene, 1997, for review), and many aspects of this reasoning are impervious to language impairments (Varley, Klessinger, Romanowski, & Siegal, 2005).

Much less is known about the construction of natural geometry, but we end with a suggestion: Natural geometry, like natural number, may be constructed by children as they discover ways to combine productively their geometric representations of large-scale layouts and small-scale objects. Moreover, these productive combinations may depend in part on widespread but culturally variable cognitive devices, including pictures, scale models, and maps.

Pictures and scale models of objects such as animals and human figures appear to be universal across human cultures, ancient and modern. Moreover, the ability to recognize objects in pictures is present early in human infancy (e.g., DeLoache, Strauss, & Maynard, 1979; Dirks & Gibson, 1977) and develops even in children with no prior experience viewing pictures (Hochberg & Brooks, 1962). By 7 months of age, infants also recognize scale models of artifacts and people (e.g., toy cars and dolls; Mandler & McDonough, 1996; Rakison, 2003), and by the end of the first year, infants begin to appreciate their symbolic functions, albeit with occasional errors (DeLoache, Uttal, & Rosengren, 2004). In contrast, the use of pictures and models to represent large-scale spatial layouts is not evident in the traces of ancient human cultures. It varies widely both over historical time and across cultures (Hagen, 1980) and begins to be mastered by children only in the third year of life (DeLoache, 1987), despite massive exposure to pictured scenes in children's books. Prior to 2.5 years of age, children can learn the positions of objects in a photograph of a room, a small-scale model of the room, or in the room itself, but they do not readily transfer information from a picture or model to the layout that it represents.

Why do children come to understand pictures or models of *objects* so much earlier than pictures or models of *large-scale spatial layouts*? The latter representations provide information about places and their relationships in the navigable layout, but they are strikingly different from the layouts that they represent. Whereas the environment is stable, with fixed cardinal directions, these representations are movable. As they are transported, pictures, models, and maps change their orientation and position with respect to the layout with every change in the navigator's position and heading. Moreover, the environment is large and surrounds the navigator, but pictures, models, and maps typically are orders of magnitude smaller and present an array located outside the space that the navigator occupies.

In the context of the findings reviewed in this article, navigating through the layout by means of pictures, models, or maps poses an additional problem. The geometric information in the map itself falls in the domain of the core system of form perception, which extracts information about distance and angle that applies to both pictures and 3D manipulable objects. In contrast, the geometric information in the navigable layout falls in the domain of the core system of navigation, which extracts information about distance and direction from 3D surface layouts. The evidence for two distinct, core systems of geometry therefore could help to explain why the practice of making pictures and models of the large-scale spatial layout is more recent, more culturally variable, and later to develop in children than the practice of making and interpreting pictures and models of small objects.

Once a human culture develops a technology for making pictures, models, or maps of spatial layouts, however, and once children in that culture begin to understand these technologies, children gain tools that could serve to extend their geometrical concepts. The power of such tools is especially clear in the case of a special kind of visual representation: purely geometric, 2D maps of the 3D navigable world.

6. Geometric maps

When do children first become able to use line drawings, providing purely geometric information about the spatial layout, to guide their navigation through that layout? By 4 years of age, remarkably, children have been shown to accomplish this task in two series of experiments. In one series (Huttenlocher, Newcombe, & Vasilyeva, 1999; see also Landau & Spelke, 1988), children were presented with a long, thin rectangular space (a sandbox) and a purely geometric map of the space (a line drawing showing an overhead view of the sandbox at about 1/8th of the scale), positioned so that the child could view both arrays at once at the same orientation. On two training trials, children were taught the correspondence between a horizontal position indicated on the map and the horizontal location of an object hidden in the sand. With no further instruction or feedback, 4-year-old children subsequently used the map to guide their search for objects hidden in the sandbox, showing sensitivity to one-dimensional distance information in the map. Performance suffered, however, when children in later experiments were required to extract positional information that varied on two dimensions (Vasilyeva & Huttenlocher, 2004). Children's difficulties in the latter experiments may stem in part from the use of an object retrieval task, which places memory demands on children (see Huttenlocher, Vasilyeva, Newcombe, & Duffy, 2008). In any case, 4-year-old children reliably extracted distance information from maps under these presentation and training conditions.

More recent studies from our laboratory reveal further abilities to use geometric maps at this age, when maps and arrays are presented separately and at different orientations, in children who are given no instruction or feedback. Shusterman, Lee, and Spelke (2008; see also Vasilyeva & Bowers, 2006) presented children with a large array of three solid objects arranged in a line or triangle and a 2D map composed of three circles in a similar geometric arrangement at 1/12th the size of the array (Fig. 5A). After showing the child an array, the experimenter presented the corresponding map, pointed at a single location on the map, and asked the child to place an object at that location in the array. Throughout the presentation of the map, the child stood with his or her back to the array, such that the map and the array were never visible at the same time. Moreover, the map was presented at a fixed orientation, either 0° or 180° offset from the array (and therefore at an orientation that preserved, respectively, the allocentric or egocentric spatial relationships between the map and the array); children were not allowed to rotate or displace the map so as to compare it to the depicted array directly. Finally, children were given no instruction to attend to the geometric relationships and no feedback on their performance; they were praised regardless of where they placed the object.

Despite the stringency of this test, 4-year-old children's performance was systematic and clearly reflected their use of distance relationships in the map (Fig. 5B). When the objects

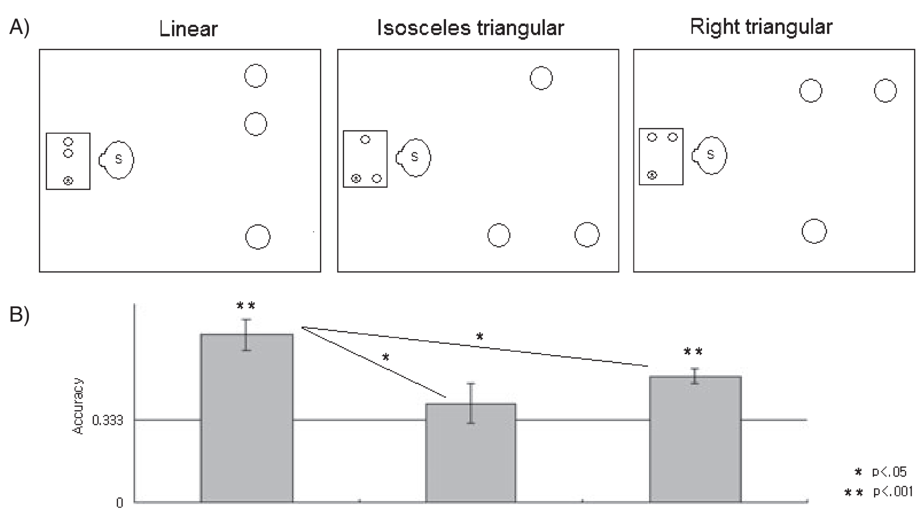

Fig. 5. Displays and performance in a test of navigation by purely geometric maps. (A) The experimental room, child (s), map, and array of three containers. Although allocentric and egocentric map orientations were used in the experiment and children were tested with multiple target locations, only the allocentric orientation, and a single target location, is depicted for each condition. (B) Children's performance in each type of array. Children performed reliably above chance on the linear array, providing evidence that they were able to use distance to distinguish between locations. Furthermore, children performed better on the linear than the triangular arrays, showing that adding angle to distance information failed to enhance their performance. In the isosceles array, children chose indiscriminately between the two mirror-image locations, indicating that they did not use sense to distinguish between locations in this task; their above-chance performance reflects their ability to distinguish the most distant corner from the other two locations.

were arranged in a line at unequal intervals, children successfully placed a toy on the correct object regardless of which object was the target, in accord with their relative distances. When the objects were arranged in a triangular array with three distinct distance and angular relationships between locations, children again used their relative distances to locate the target object and performed most accurately when the target was the most distant object in the array. However, children performed no better on the triangular than on the linear or the isosceles array, suggesting that the addition of distinctive angle information did not enhance their performance. Finally, when the objects were arranged in an isosceles triangular array, children again used the distance information to place objects successfully at the most distant target location, but they failed to use directional information to distinguish the two remaining locations.

At 4 years of age, therefore, children spontaneously extracted distance from both the 2D map and the 3D layout that it represented, and they used the common distance relations in the two arrays to navigate by the map. Nevertheless, children of this age did not show a clear ability to use angle or directional information from the map to guide their behavior in the layout. Research on children's core systems of geometry may explain both this pattern of performance and the performance of children in the earlier map studies (Huttenlocher et al., 1999; Vasilyeva & Bowers, 2006), because *distance* is the only source of information that children readily extract from both small visual forms and large navigable layouts. The abilities manifest in this experiment may represent an early step in children's construction of a more general, unified system of Euclidean geometry.

Might full Euclidean geometry be constructed from these beginnings? To address this question, it is necessary to study the subsequent development of children's abilities to extract geometric information from maps. Although there is a wealth of research on the mapping abilities of children (e.g., Davies & Uttal, 2007) and adults (e.g., Golledge, 2008), there is little research that focuses on purely geometric maps, and that tests systematically for sensitivity to distance, angle, and directional information. Nevertheless, a pattern may be emerging from the few studies that have teased these relationships apart. At about 6 years of age, children extract angle as well as distance information from maps, even though they continue to fail to extract directional informational (Dehaene et al., 2006; E. S. Spelke, C. Gilmore, & S. McCarthy, unpublished data), even though they continue to fail to extract directional information (Gilmore et al., unpublished data; Dehaene et al., 2006). Adults, in contrast, extract information about distance, angle, and direction, although they continue to make more errors with directional information than with information about distance or angle (Dehaene et al., 2006).

Although much more research is needed, these findings suggest that children and adults enhance their geometric representations in two ways. First, through their experience with pictures, scale models, and maps, children may begin to view large-scale layouts not only as navigable surroundings but also as visual displays with forms that have distinctive angular shapes. Because children come to understand the symbolic functions of pictures, models, and maps over the first 4 years of life, children can begin to add angle information to their representations of large-scale layouts. Second, through their experience with physical and mental rotation, children and adults may become able to treat small-scale objects and forms not only as visual displays with distinctive shapes but as layouts that can be explored from different perspectives, by means of navigation systems that allow for stable representations of the distinction between leftward and rightward directions (see Landau & Lakusta, 2009). By applying two different kinds of geometrical analysis to the same arrays, children may therefore discover new relations between Euclidean distances, angles, and directions. By extending each of these kinds of geometrical analysis to new types of arrays, moreover, children may develop geometrical concepts that are more abstract and general than the concepts provided by their core systems.

7. Overview

Carey's program of research sets three tasks for the study of cognitive development. The first task is to specify the core systems that provide the conceptual primitives on which later systems of knowledge build, by drawing on comparative research at the time scales of human evolution, human historical and cultural development, and human ontogeny. The second task is to describe the conceptual changes that occur over the course of development, characterizing both the common and the divergent concepts of younger and older children. The third task is to characterize the processes of conceptual change that cause the emergence of the older child's system of concepts from the concepts and cognitive resources of the younger child.

The study of the development of geometrical concepts has begun these tasks. Research on young human children, nonhuman animals, and human adults in diverse cultures provides evidence for at least two core systems of geometry that are present and functional early in human development, that predate the evolution of humans as a species, and that remain universally present in human adults. Research on older children provides evidence for the emergence of capacities to relate these systems. At 4 years, children appear to relate the shapes of objects and of the surface layout only with respect to their common *distance* relations. With development, however, children also relate these representations on the basis of *angle*, and by adulthood, *direction*. We have learned little, thus far, about the processes by which this integration occurs. What leads the child to view the navigable surface layout as a large-scale form, with the angular relationships that apply to visible objects? And what leads the child to view small visible forms as an array of paths and places to which one can navigate in one's mind? These questions cannot now be answered, but Carey's theoretical tools, together with the broad array of empirical methods now probing geometrical abilities, should allow investigators to ask them.

Notes

1. Two active questions in this field concern the conditions under which disoriented children and animals use nongeometric properties of surfaces to guide their navigation, and the relationship between the navigation processes that respond to geometric and

to nongeometric information (see Cheng & Newcombe, 2005; Lee, Shusterman, & Spelke, 2006; Newcombe & Ratliff, 2007; Shusterman & Spelke, 2005). Because the present article concerns only navigation by layout geometry, we do not consider these questions here.

2. Readers may wonder why 2-year-old children reoriented in accord with the differences in wall brightness but not color, and in accord with the relative sizes but not shapes of visual forms. One possibility, suggested by Huttenlocher and Lourenco (2007), is that reorientation is effective when children detect an ordering of continuously varying dimensions of any kind, whether spatial (size) or nonspatial (brightness). A second possibility is that the relative brightness of walls or size of their texture elements conveyed an impression of depth and led children to misperceive the square room as slightly rectangular. Consistent with the latter possibility, we recently found that children reoriented themselves by the shape of a rectangular enclosure even when its walls differed only slightly in length (in an 8:9 ratio; S. A. Lee, N. Winkler-Rhoades, and E. Spelke, unpublished data). When the large and small dot patterns of Huttenlocher and Lourenco (2007) were placed on the walls of such a room, moreover, children reoriented successfully if the larger dots appeared on the nearer sides, but not if the smaller dots appeared on the nearer sides, providing evidence that the dot patterns function as a depth cue.
3. Hupbach and Nadel (2005) reported that children begin to use the shape of the rhombus at about 4 years of age, but this finding does not reveal whether 4-year-old children reorient by angle information. Because a complete rhombus differs from a square both in angle (squares have four equal angles, rhombuses have two pairs of distinct angles) and in aspect ratio (a rhombic room, like a rectangular room, has major and minor axes of distinctive relative lengths), 4-year-old children could reorient by either of these properties. Recent experiments that teased apart these two variables provide evidence that 2-year-old children use aspect ratio but not angle to reorient (S. A. Lee & E. S. Spelke, unpublished data). Whatever the status of angle at 4 years of age, however, Hupbach and Nadel's experiments show that younger children fail to reorient by detecting the angles at which extended surfaces meet.

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