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Brief report

Children's expectations about training the approximate number system

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Humans possess a developmentally precocious and evolutionarily ancient approximate number system (ANS) whose sensitivity correlates with uniquely human symbolic arithmetic skills. Recent studies suggest that ANS training improves symbolic arithmetic, but such studies may engender performance expectations in their participants that in turn produce the improvement. Here, we assessed 6- to 8-year-old children's expectations about the effects of numerical and non-numerical magnitude training, as well as states of satiety and restfulness, in the context of a study linking children's ANS practice to their improved symbolic arithmetic. We found that children did not expect gains in symbolic arithmetic after exercising the ANS, although they did expect gains in ANS acuity after training on any magnitude task. Moreover, children expected gains in symbolic arithmetic after a good night's sleep and their favourite breakfast. Thus, children's improved symbolic arithmetic after ANS training cannot be explained by their expectations about that training.

Investigators from diverse areas in the cognitive sciences have proposed causal links between the processes that engage approximate numerical representations and those that underlie performance on tests of symbolic mathematics (e.g., Dehaene, 2011). The evidence supporting such claims begins with the widespread findings that the acuity with which an individual can employ their approximate number system (ANS) to distinguish between arrays of objects based on their number correlates with performance on tests of symbolic mathematics (e.g., Chen & Li, 2014; Halberda, Mazzocco, & Feigenson, 2008; Hornung, Schiltz, Brunner, & Martin, 2014; Starr, Libertus, & Brannon, 2013).

Recent cognitive training studies have suggested a causal link between ANS practice and improved performance on tests of symbolic mathematics: Both adults and children perform exact, symbolic arithmetic faster and more accurately after training on approximate, non-symbolic, representations of number, relative to training on other magnitude tasks (Hyde, Khanum, & Spelke, 2014; Kuhn & Holling, 2014; Park & Brannon, 2013). The causal conclusions of such training studies appeal to a broad audience,

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including psychologists who are interested in their basic scientific claims and educators who may apply their methods to teaching mathematics in school. Boot, Simons, Stothart, and Stutts (2013) argue, however, that cognitive training studies with active control conditions, presenting tasks with the same structure but with different content, are susceptible to placebo effects: Participants may expect training to yield benefits on assessments with similar content, and these expectations, rather than any gains in competence, may account for performance differences between the experimental and active control groups. ANS training experiments could well be vulnerable to such placebo effects. Participants may expect benefits on assessments with numerical content after training on an approximate numerical task, relative to training on tasks with no numerical content. Previous ANS training experiments thus risk yielding misleading findings about the causal link between the ANS and symbolic arithmetic.

Here, we ask whether children's expectations could drive the effects of ANS training on symbolic arithmetic. More specifically, does training approximate, non-symbolic numerical skills affect elementary school children's performance on exact, symbolic mathematics simply because those who received this training approach a symbolic mathematics assessment with different expectations than those who received the control training? This question is important both for developing accurate theories of numerical development and for teaching mathematics in school. If such expectation effects can be ruled out, then existing ANS training studies hold promise for revealing the causal links between the intuitive and early emerging representations of number and the uniquely human formal systems supporting symbolic mathematics. In contrast, if children's expectations account for the effects of ANS training observed in past experiments, it will be critical to craft new cognitive training experiments that control for such effects.

Training experiments with children may carry heightened risks of expectation effects because children's motivations and expectations play a central role in their school achievement (Dweck, 1999). In addition, elementary school children have metacognitive awareness of their numerical discrimination judgments, and this awareness predicts their performance on the mathematics learned in school (Vo, Li, Kornell, Pouget, & Cantlon, 2014). In the present study, we thus examined 6- to 8-year-old children's expectations in the context of a recent short-term training experiment linking children's non-symbolic numerical practice to their performance on symbolic mathematics (Hyde *et al.*, 2014, Experiment 1).

In the experiment by Hyde *et al.* (2014), separate groups of children were given about 15 min of training on one of four approximate, non-symbolic magnitude tasks. One training task focused on adding arrays of dots; another task focused on comparing arrays of dots; a third task focused on adding line lengths; and a fourth task focused on comparing objects on the basis of surface brightness (Figure 1). Then, children completed a test of written symbolic addition in Arabic notation and of non-symbolic numerical acuity (comparing pairs of dot arrays based on number, after Halberda *et al.*, 2008). On the symbolic addition test, children who were trained on one of the two ANS training tasks (adding or comparing dot arrays) performed faster (with no loss in accuracy) than those who were trained on line length addition or surface brightness comparison. In addition, those trained on numerical comparison of dot arrays performed more accurately than those trained on line length addition. In contrast, ANS-trained children showed no gains in non-symbolic numerical acuity compared to those trained on non-numerical content. These results provide evidence for enhanced exact symbolic arithmetic after approximate, non-symbolic number system training. Because this study employed active control groups trained on non-numerical content, however, we investigated whether

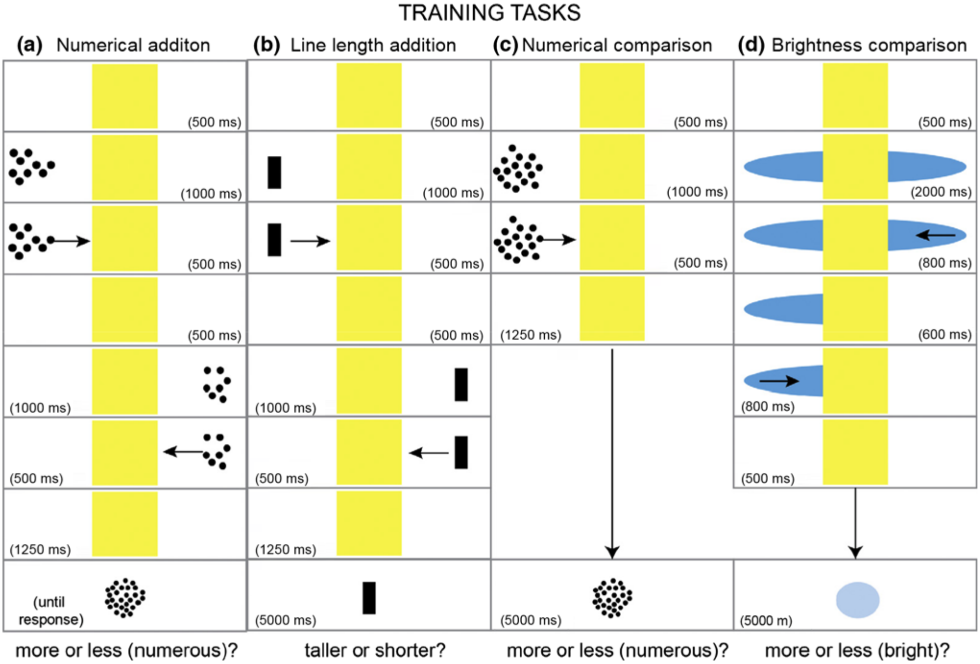

Figure 1. Reprinted with permission from Hyde *et al.* (2014), in which children were presented with one of four non-symbolic training tasks. Two of these tasks presented numerical content [(a) Numerical Addition and (c) Numerical Comparison] and two of these tasks presented non-numerical content [(b) Line Length Addition and (d) Brightness Comparison]. The sequences of events in a sample training trial from each task are organized vertically from start (top) to finish (bottom) in the figure. The horizontal arrows indicate the movement of an item in the visual display.

children's expectations about each type of training could account for the obtained training outcomes.

Methods

Twenty-four 6- to 8-year-old children (as in Hyde *et al.*, 2014; mean = 7 years 5 months, range 6 years 10 months to 7 years 10 months) participated in this study. Three more children were excluded because of symbolic addition scores more than four standard deviations below the mean obtained by Hyde *et al.* (2014). Children first completed portions of the outcome assessments used by Hyde *et al.* (2014) in a fixed order: They solved twenty 1- and 2-digit symbolic addition problems (half of the original assessment in Hyde *et al.*, 2014) and completed 60 items from a non-symbolic numerical comparison test (the complete assessment from Hyde *et al.*, 2014), probing their sensitivity to numerical differences between pairs of dot arrays at ratios of 2:1, 3:2, 4:3 and 6:5 (see Hyde *et al.,* 2014 for further details).

In counterbalanced order, children were then introduced to each of the four training tasks in Hyde *et al.* (2014). To give them a good idea of each training task, they were presented with eight problems for each task from the original study. The first four were from the practice problems and the second four were from the training itself. After each training task was introduced, children saw a figure depicting that training task and a figure depicting the symbolic addition test. They were asked whether, after playing that training game for a while, they thought that their performance on the addition test would change, relative to their initial performance. Specifically, children were asked whether they thought they would get more, fewer, or the same number of questions right and whether they would answer more quickly, more slowly, or in the same amount of time. Then, children were shown a figure depicting the non-symbolic numerical acuity test and were asked the same questions about their expected accuracy and speed after performing the training task. Finally, children were asked in counterbalanced order to consider these same questions after a great night's sleep and their favourite breakfast, or after little sleep and no breakfast. Children's responses were recorded on a three-levelled ordinal scale.

Results

Mirroring the analyses of other investigations probing placebo effects in cognitive training studies (e.g., Boot *et al.*, 2013), we first compared children's expectations about the effects of training approximate numerical addition and approximate numerical comparison (both ANS training tasks), as well as line length addition and brightness comparison (both non-ANS training tasks) to the specific outcomes obtained in Hyde *et al.*'s (2014) experiment. None of children's expectations aligned with the previously observed ANS training effects: In particular, children did not expect that their speed and accuracy on Hyde *et al.*'s (2014) test of symbolic addition would improve more after training on ANS tasks than after training on non-numerical magnitude tasks (Table 1).

Nevertheless, children judged that they would perform both tests more accurately and faster after a good night's sleep and their favourite breakfast than after little sleep and no breakfast (Table 2). Such a result reveals that although children's expectations about the effects of ANS training on symbolic arithmetic did not align with Hyde *et al.*'s (2014) training outcomes, they appeared to understand the questions, were motivated to answer them, and formed clear expectations about the effects of satiety and restfulness on their test performance.

Table 1. Performance on the outcome measures that yielded differences in Hyde *et al.*'s (2014) Experiment 1, compared to children's expectations about their efficacy in the present study

Training task	Test	Measure	Hyde *et al.* (2014)	Expectations
Numerical comparison	Symbolic addition	Speed	$t(46) = 2.53$,	McNemar = 2.87,
Brightness comparison	Symbolic addition	Speed	$p < .05$	$p = .413$
Numerical comparison	Symbolic addition	Speed	$t(46) = 2.33$,	McNemar = 2.14,
Line length addition	Symbolic addition	Speed	$p < .05$	$p = .543$
Numerical addition	Symbolic addition	Speed	$t(46) = 2.18$,	McNemar = 2.14,
Brightness comparison	Symbolic addition	Speed	$p < .05$	$p = .543$
Numerical addition	Symbolic addition	Speed	$t(46) = 2.03$,	McNemar = 0.53,
Line length addition	Symbolic addition	Speed	$p < .05$	$p = .912$
Numerical comparison	Symbolic addition	Accuracy	$t(46) = 2.58$,	McNemar = 1.00,
Line length addition	Symbolic addition	Accuracy	$p < .05$	$p = .801$

Table 2. Children's expectations about the effects of different states of satiety and restfulness on Hyde *et al.*'s (2014) tests of symbolic addition and non-symbolic numerical acuity

State of satiety and restfulness	Test	Measure	Expectations
Good night's sleep, favourite breakfast	Symbolic addition	Accuracy	McNemar = 16.33, p = .001
Poor night's sleep, no breakfast	Symbolic addition	Accuracy	
Good night's sleep, favourite breakfast	Symbolic addition	Speed	McNemar = 9.50, p = .023
Poor night's sleep, no breakfast	Symbolic addition	Speed	
Good night's sleep, favourite breakfast	Non-symbolic numerical acuity	Accuracy	McNemar = 14.00, p = .003
Poor night's sleep, no breakfast	Non-symbolic numerical acuity	Accuracy	
Good night's sleep, favourite breakfast	Non-symbolic numerical acuity	Speed	McNemar = 13.80, p = .003
Poor night's sleep, no breakfast	Non-symbolic numerical acuity	Speed	

We further analysed children's expectations for each training task, test administered, and outcome measure (accuracy or speed) from Hyde *et al.* (2014) using an ordinal logistic regression mixed model. (These data met the specifications of having proportional odds at each level of the ordinal scale: Likelihood ratio test: p = .112.) Children showed no differential expectations about the effects of the four training tasks. However, across training tasks, they predicted greater gains in non-symbolic numerical acuity than in symbolic addition, and in accuracy than in speed (Figure 2; Table 3). In other words, children expected improvement on a measure of ANS acuity after training on any approximate magnitude task, whether or not that task specifically targeted the numerical representations that underlie the ANS. In addition, children did not expect that magnitude training would transfer to tests of symbolic arithmetic. These expectations contrast further with the obtained training effects of Hyde *et al.* (2014): Only children trained on ANS tasks (i.e., dot addition or comparison) showed training gains relative to those trained on non-ANS tasks (i.e., line length addition or brightness comparison); these gains were shown on the test of symbolic addition but not numerical acuity; and they were manifest by gains in speed but not accuracy (except in one instance; Table 1).

Discussion and conclusions

The present study revealed that children's expectations concerning the effects of ANS training tasks relative to non-numerical training tasks are unlikely to account for the findings of Hyde *et al.* (2014): Six- to eight-year-old children did not expect that ANS training would improve their subsequent performance on Hyde *et al.*'s (2014) test of symbolic arithmetic more than non-numerical training. Nevertheless, children expected differential gains in symbolic arithmetic given different states of satiety and restfulness,

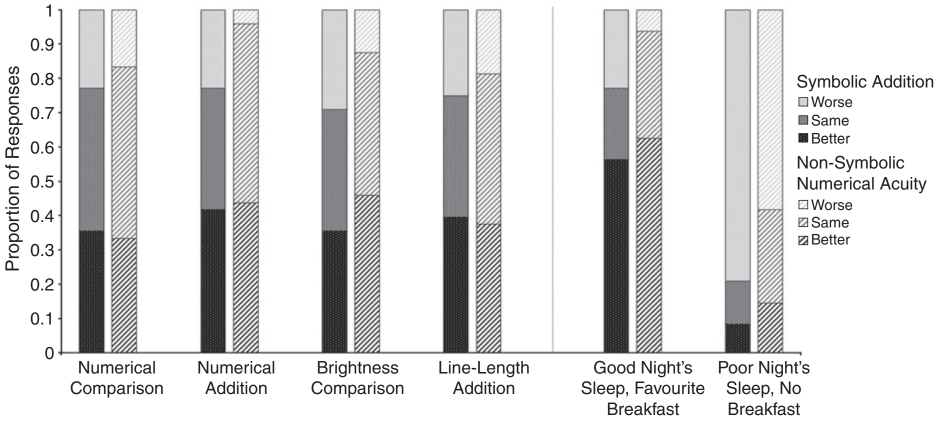

Figure 2. The proportion of children's responses to whether they would perform better, the same, or worse on Hyde *et al.*'s (2014) symbolic addition assessment and assessment of non-symbolic numerical acuity after training on the four non-symbolic magnitude tasks and given different states of satiety and restfulness.

Table 3. The ordinal logistic regression mixed model evaluating children's expectations across pairs of the following: Training conditions, tests, and measures

	Reference	Comparison	*p*-Value[a]	% Change in odds ratio	95% CI for % change
Training	Numerical comparison	Numerical addition	.720	55.81[b]	10.86[c]–172.33[b]
	Numerical comparison	Brightness comparison	1.000	14.06[b]	34.60[c]–98.90[b]
	Numerical comparison	Line length addition	1.000	7.33[b]	38.55[c]–87.49[b]
	Numerical addition	Brightness comparison	1.000	26.80[c]	58.21[c]–28.24[b]
	Numerical addition	Line length addition	.970	31.11[c]	60.74[c]–20.88[b]
	Brightness comparison	Line length addition	1.000	5.89[c]	46.28[c]–64.85[b]
Test	Symbolic addition	Non-symbolic numerical acuity	.031	35.54[b]	4.04[b]–56.70[b]
Measure	Speed	Accuracy	.008	41.66[b]	13.09[b]–60.85[b]

Note. [a]Holm corrected for multiple comparisons. Percentage changes in the proportional odds ratios produced by the model are included to quantify the differences in which children expected that the odds for improving would be [b]greater for the comparison group than the reference group or the [c]reverse. For example, children expected that after training, the odds of improving on the non-symbolic numerical acuity test were 35.54% more likely than the odds of improving on the symbolic addition test. *p*-values and 95% confidence intervals are provided for interpreting these differences.

and in general, children expected greater gains in ANS acuity than in symbolic arithmetic after training on any of Hyde *et al.*'s (2014) magnitude tasks. By showing that children's expectations about ANS training do not align with training outcomes, we thus address an alternative account based on expectations that would challenge a causal link between the ANS and symbolic mathematics.

Our findings also underscore the importance of testing more broadly for placebo effects in cognitive training studies with children: The children in our sample displayed a positive pattern of expectations about how states of satiety and restfulness might affect their performance on Hyde *et al.*'s (2014) assessments and how magnitude training in general might affect their performance on an outcome measure with a similar structure. These findings, in turn, accord with a wealth of evidence that children of this age gain increasing awareness both of thinking in general (e.g., Flavell, Green, Flavell, Harris, & Astington, 1995) and of their sensitivity to number in particular (Vo *et al.*, 2014). The growing body of work relying on cognitive training studies with children will benefit from further elucidating the expectations and motivations that particular training methods engender (Barner *et al.*, in press; Cheng & Mix, 2014; Goldin *et al.*, 2014; Kuhn & Holling, 2014), and the present study offers one method for doing so.

The present findings are also consistent with the claim that our developmentally precocious and evolutionally ancient numerical sensitivity is causally linked to the uniquely human and culturally constructed formal system of mathematics that supports human knowledge at its highest reaches. Nevertheless, the robustness and generality of ANS training effects merit further study with adults and children of different ages through training experiments that assess and, if necessary, control for the effects of participants' expectations on their cognitive performance.

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